

CONTROLLING DECOHERENCE OF TRANSPORTED QUANTUM SPIN INFORMATION IN SEMICONDUCTOR SPINTRONICS

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We investigate quantum coherence of electron spin transported through a semiconductor spintronic device, where spins are envisaged to be controlled by electrical means via spin-orbit interactions. To quantify the degree of spin coherence, which can be diminished by an intrinsic mechanism where spin and orbital degrees of freedom become entangled in the course of transport involving spin-orbit interaction and scattering, we study the decay of the off-diagonal elements of the spin density matrix extracted directly from the Landauer transmission matrix of quantum transport. This technique is applied to understand how to preserve quantum interference effects of fragile superpositions of spin states in ballistic and non-ballistic multichannel semiconductor spintronic devices.

1. Introduction

The major goal of recent vigorous efforts in spintronics is to create, store, manipulate at a given location, and transport coherent electron spin states through conventional semiconductor heterostructures.¹ The two principal challenges for new generation of spintronic devices are efficient injection of spin into various semiconductor nanostructures and coherent control of spin. In particular, preserving spin coherence, which enables coherent superpositions of states $a|\uparrow\rangle + b|\downarrow\rangle$ and corresponding quantum-interference effects, is essential for both quantum computing with spin-based qubits² and plethora of the proposed classical information processing devices that encode information into electron spin.^{1,3}

The electrical control of spin via Rashba spin-orbit (SO) interaction,⁴ which arises due to inversion asymmetry of the confining electric potential for two-dimensional electron gas (2DEG), has become highly influential concept in semiconductor spintronics. A paradigmatic semiconductor spintronic device of this kind is the Datta-Das spin-field-effect transistor³ (spin-FET) where current passing through 2DEG in semiconductor heterostructure is modulated by changing the strength of Rashba SO interaction via gate electrode.⁵ The injected current can be modulated in this scheme only if it is fully polarized, while precessing spin has to remain *phase-coherent* during propagation between the two ferromagnetic electrodes. Although spin injection into bulk semiconductors has been demonstrated at low temperatures, creating and detecting spin-polarized currents in high-mobility 2DEG has turned out to be a much more demanding task.⁴

For devices pushed into the mesoscopic realm,⁶ where at low temperature $T \ll 1\text{K}$ and at nanoscales full electron quantum state $|\Psi\rangle \in \mathcal{H}_o \otimes \mathcal{H}_s$ remains pure (in the tensor product of orbital and spin Hilbert spaces) due to suppression of dephasing processes, it becomes possible to modulate even unpolarized currents. In recently proposed spintronic ring device,⁷ the conductance of unpolarized charge transport through a single channel ring can be modulated between 0 and $2e^2/h$ by changing the Rashba electric field via gate electrode covering the ring.⁵ This device exploits spin-dependent quantum interference effects involving topological phases acquired in transport through multiply-connected geometries, thereby avoiding ferromagnetic elements and spin injection problems.

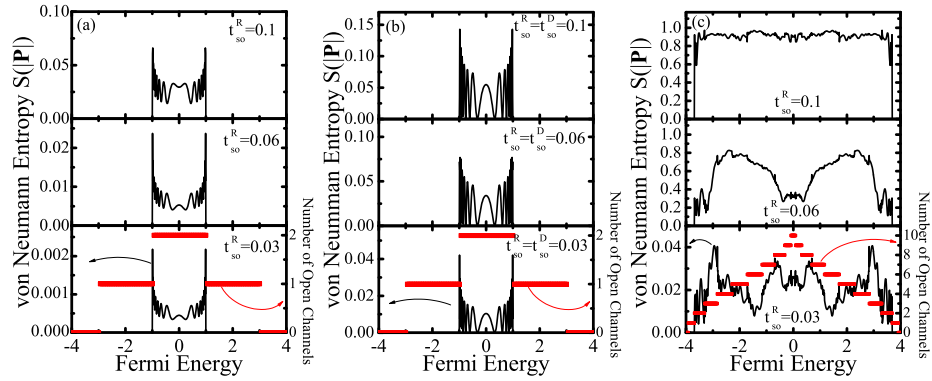


Figure 1. The von Neumann entropy of spins transmitted through a clean semiconductor nanowire supporting maximum (around the band center) of two [panels (a) and (b)] or ten [panel (c)] conducting channels. The wire is modeled by a Hamiltonian Eq. (2) with different strengths of the SO couplings t_{so}^R and t_{so}^D on lattices: (a) 2×100 , (b) 2×100 , and (c) 10×100 . Note that quantum coherence starts to decrease ($S > 0$) when the second conducting channels becomes available for quantum transport.

However, even when all other spin decoherence mechanisms due to coupling to external environment are suppressed,⁸ the same SO coupling that is envisaged to control the spin can act to entangle spin and orbital quantum states. In such cases, one cannot associate a pure state $|\sigma\rangle \in \mathcal{H}_s$ to the spin degree of freedom any more.⁹ The reduction of phase-coherence of an open spin quantum system is formally described (as is the case of any decoherence process⁸) as the decrease of the off-diagonal elements of a two-level system density matrix $\hat{\rho}_s = (1 + \mathbf{P} \cdot \hat{\sigma})/2$, where $\mathbf{P} = (P_x, P_y, P_z)$ is the *spin polarization vector*. The decoherence increases the spin von Neumann entropy $S = -\text{Tr}[\hat{\rho}_s \log_2 \hat{\rho}_s]$, which, in the case of spin- $\frac{1}{2}$ particle, is in one-to-one correspondence with the magnitude of the spin polarization vector $|\mathbf{P}|$: $S(|\mathbf{P}|) = -(1 + |\mathbf{P}|)/2 \log_2(1 + |\mathbf{P}|)/2 - (1 - |\mathbf{P}|)/2 \log_2(1 - |\mathbf{P}|)/2$. For *pure* states, which are fully coherent by definition, the polarization vector has unit magnitude $|\mathbf{P}| = 1 \Leftrightarrow S = 0$, while $|\mathbf{P}| = 0 \Leftrightarrow S = 1$ characterizes a non-pure state

that is completely unpolarized. For $0 < |\mathbf{P}| < 1$ ($0 < S(|\mathbf{P}| < 1)$), a spin- $\frac{1}{2}$ particle is in partially coherent quantum state which is described by a *mixture* (or statistical superpositions) $\hat{\rho}_s^2 \neq \hat{\rho}_s$.

2. Spin density matrix of detected current in semiconductor nanostructures

To understand the coherence properties of transported spins (or mobile qubits¹⁰), we have developed a formalism⁹ that extracts the spin density matrix from the Landauer transmission matrix of quantum transport. The \mathbf{t} -matrix is traditionally employed⁹ to compute the spin resolved conductances $G^{\sigma'\sigma} = e^2/h \sum_{n'n} |\mathbf{t}_{n'n,\sigma'\sigma}|^2$. In the case of spin-dependent transport, the Landauer \mathbf{t} -matrix, which defines the outgoing asymptotic scattering state in the left right lead of a two-probe device $|\text{out}\rangle$ when electron is injected in conducting channel $|n\rangle$ with spin $|\sigma\rangle$, also encodes the entanglement of orbital conducting channels (i.e., transverse propagating modes defined by the leads in the scattering picture of quantum transport⁶) and spin. This is due to the fact that $|\text{out}\rangle = \sum_{n',\sigma'} \mathbf{t}_{n'n,\sigma'\sigma} |n'\rangle \otimes |\sigma'\rangle$ is, in general, a non-separable state (i.e., a sum of the tensor product states $|n\rangle \otimes |\sigma\rangle$ that define spin-polarized conducting channels) because of spin-momentum entanglement¹¹ generated by spin-independent scattering (off lattice imperfections, phonons, nonmagnetic impurities, interfaces, ...) in the presence of SO interaction.^a

By viewing the current in the right lead of a two-probe spintronic device as an ensemble of *improper* mixtures, each of which is generated after injecting electrons in different spin-polarized channels $|n\rangle \otimes |\sigma\rangle$ and propagating them through complicated semiconductor environment, we introduce a *spin density matrix of the detected current*⁹

$$\hat{\rho}_c = \frac{e^2/h}{n_{\uparrow}(G^{\uparrow\uparrow} + G^{\downarrow\uparrow}) + n_{\downarrow}(G^{\uparrow\downarrow} + G^{\downarrow\downarrow})} \times \sum_{n',n=1}^M \begin{pmatrix} n_{\uparrow}|\mathbf{t}_{n'n,\uparrow\uparrow}|^2 + n_{\downarrow}|\mathbf{t}_{n'n,\uparrow\downarrow}|^2 & n_{\uparrow}\mathbf{t}_{n'n,\uparrow\uparrow}\mathbf{t}_{n'n,\downarrow\uparrow}^* + n_{\downarrow}\mathbf{t}_{n'n,\uparrow\downarrow}\mathbf{t}_{n'n,\downarrow\downarrow}^* \\ n_{\uparrow}\mathbf{t}_{n'n,\uparrow\uparrow}^*\mathbf{t}_{n'n,\downarrow\uparrow} + n_{\downarrow}\mathbf{t}_{n'n,\uparrow\downarrow}^*\mathbf{t}_{n'n,\downarrow\downarrow} & n_{\uparrow}|\mathbf{t}_{n'n,\downarrow\uparrow}|^2 + n_{\downarrow}|\mathbf{t}_{n'n,\downarrow\downarrow}|^2 \end{pmatrix}. \quad (1)$$

Here the injected current is assumed to be in the most general (i.e., partially polarized) state $\hat{\rho}_s = n_{\uparrow}|\uparrow\rangle\langle\uparrow| + n_{\downarrow}|\downarrow\rangle\langle\downarrow|$. Special cases of injection of 100% spin- \uparrow polarized or spin- \downarrow polarized current correspond to $n_{\uparrow} = 1, n_{\downarrow} = 0$ and $n_{\uparrow} = 0, n_{\downarrow} = 1$, respectively. The spin polarization vector of the current is $(P_x, P_y, P_z) = \text{Tr}[\hat{\rho}_c \hat{\boldsymbol{\sigma}}]$, and $\hat{\rho}_c$ also specifies the von Neumann entropy $S(|\mathbf{P}|)$ of the ensemble of transported

^aThe entanglement of spin and orbital degrees of freedom⁹ is somewhat different¹¹ from the familiar entanglement between different particles that can be widely separated and utilized for quantum communication,^{8,11} because both degrees of freedom belong to the same particle. Formally similar entanglement of different degrees of freedom of one and the same particle has been pursued recently in Ref.12—an entanglement of a transverse wave function $|\Phi_n\rangle$ and a plane wave $|k\rangle$ (with \mathbf{k} -vector along the direction of transport) in the outgoing lead that form the basis of orbital conducting channels $|n\rangle = |\Phi_n\rangle \otimes |k\rangle$.

spins that comprise the current. Thus, the equations for (P_x, P_y, P_z) , together with the Landauer formula for spin-resolved charge conductances,⁹ provide complete description of coupled spin-charge transport in finite-size devices while intrinsically handling relevant boundary conditions.

We model generic semiconductor nanostructure by a single-particle Hamiltonian

$$\hat{H} = \left(\sum_{\mathbf{m}} \varepsilon_{\mathbf{m}} |\mathbf{m}\rangle \langle \mathbf{m}| - t \sum_{\langle \mathbf{m}, \mathbf{m}' \rangle} |\mathbf{m}\rangle \langle \mathbf{m}'| \right) \otimes \hat{I}_s + \frac{\alpha \hbar}{2a^2 t} (\hat{v}_y \otimes \hat{\sigma}_x - \hat{v}_x \otimes \hat{\sigma}_y) + \frac{\beta \hbar}{2a^2 t} (\hat{v}_x \otimes \hat{\sigma}_x - \hat{v}_y \otimes \hat{\sigma}_y), \quad (2)$$

written in the local- s -orbital \otimes spin basis on the lattice $M \times L$ (the hopping t between the orbitals sets the unit of energy), where $(\hat{v}_x, \hat{v}_y, \hat{v}_z)$ is the velocity operator and we utilize the tensor product \otimes of operators in $\mathcal{H}_o \otimes \mathcal{H}_s$. The second term in the Hamiltonian is the Rashba SO interaction (whose electric field lies along the z -axis, while effective \mathbf{k} -dependent magnetic field $\mathbf{B}(\mathbf{k})$ of the SO interaction $\mathbf{B}(\mathbf{k}) \cdot \hat{\boldsymbol{\sigma}}$ emerges in the xy -plane), while the third term is the Dresselhaus one (arising from the bulk inversion asymmetry). In the ballistic wires of Sec. 3, the on-site potential energy is $\varepsilon_{\mathbf{m}} = 0$. The disorder in Sec. 4 is introduced through a standard random variable $\varepsilon_{\mathbf{m}} \in [-W/2, W/2]$. The spin-resolved transmission matrix, $\mathbf{t} =$

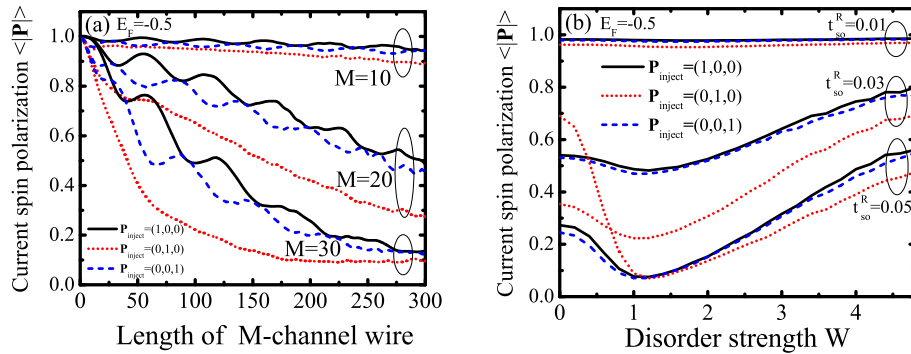


Figure 2. Quantum coherence of spins (injected with different direction of \mathbf{P}) transmitted through M -channel wires as a function of the wire width [panel (a)] or the strength of the disorder W and the Rashba SO coupling t_{so}^R . The wires are modeled by the Hamiltonian in Eq. (2) on the lattice $M \times L$ [$\equiv 30 \times 100$ in panel (b)].

$2\sqrt{-\text{Im} \hat{\Sigma}_L^r \otimes \hat{I}_s \cdot \hat{G}_{1N}^r \cdot \sqrt{-\text{Im} \hat{\Sigma}_R^r \otimes \hat{I}_s}}$ is obtained from the real \otimes spin space Green function $\hat{G}^r = [E\hat{I}_o \otimes \hat{I}_s - \hat{H} - \hat{\Sigma}^r \otimes \hat{I}_s]^{-1}$. Here $\hat{\Sigma}^r$ is the self-energy introduced by the leads,⁶ and \hat{I}_o and \hat{I}_s are the unit operators in \mathcal{H}_o and \mathcal{H}_s , respectively. The matrix elements of the SO terms in Eq. (2) contain "SO hopping parameters" $t_{\text{so}}^R = \alpha/2a$ and $t_{\text{so}}^D = \beta/2a$ that set the energy scales of the Rashba and Dresselhaus

coupling, respectively.^b

3. Spin coherence in ballistic spin-FET-type devices

Despite advances in nanofabrication technology, it is still a challenge to fabricate semiconductor nanowire that contains only one transverse propagating mode. We investigate spin coherence in multichannel *clean* wires in Figure 1, which plots the spin entropy $S(|\mathbf{P}|)$ as a function of the Fermi energy E_F of electrons whose transmission matrix $\mathbf{t}(E_F)$ determines coupled spin-charge transport in a wire supporting at most two or ten orbital conducting channels. The current injected from the left

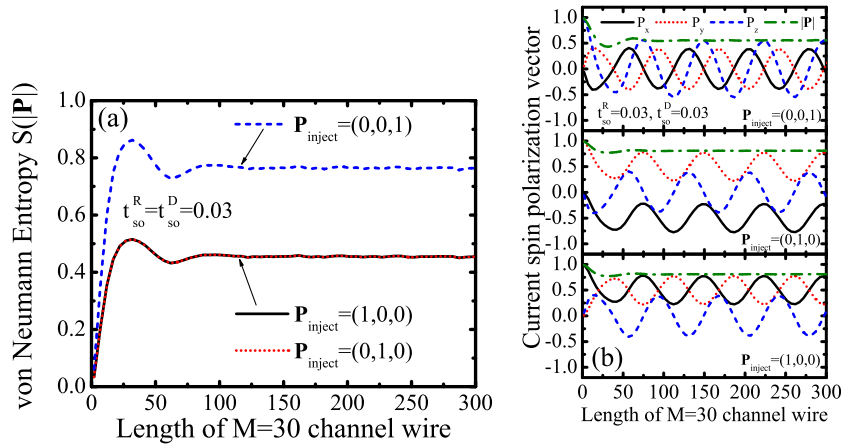


Figure 3. Spin coherence along the 2DEG region, modeled on the lattice $30 \times L$, of a non-ballistic spin-FET device¹⁴ (where $t_{\text{so}}^R = t_{\text{so}}^D$) for different directions of polarization of injected current.

lead is assumed to be fully polarized along the direction of transport (the x -axis chosen here), as in the spin-FET operation where such setup ensures high level of current modulation.³ As long as only one conducting channel is open, spin remains coherent since outgoing state in the right lead is $(a|\uparrow\rangle + b|\downarrow\rangle) \otimes |n=1\rangle$. At exactly the same E_F where the second channels $|n=2\rangle$ becomes available for transport, $S(|\mathbf{P}|)$ becomes non-zero signaling that spin state loses its purity. This is due to the fact that at this E_F , the quantum state of transported spin becomes *entangled* to its orbital state,⁹ $|\text{out}\rangle = a|\nearrow\rangle \otimes |e_1\rangle + b|\swarrow\rangle \otimes |e_2\rangle$. The scattering at the lead-semiconductor interface, which in the presence of the SO interaction generates such non-separable (i.e., entangled) outgoing state is induced by different nature of electron states in the wire and in the leads. While recent studies¹³ have pointed out that for *strong* Rashba SO interaction this effect can lead to complete suppression

^bIn current experiments⁵ maximum achieved values of t_{so}^R are of the order of $\sim 0.01t$.

of spin injection, here we unearth how even moderate Rashba coupling in wires of few nanometers width can affect coherence of ballistically transported spins. This becomes increasingly detrimental when more channels are opened, as demonstrated in Fig. 1 for a $M = 10$ channel nanowire.

4. Spin coherence in non-ballistic spin-FET-type devices

When transported charge scatters off spin-independent impurities in 2DEG with SO interaction, its spinor will get randomized and the disorder-averaged current spin polarization $\langle |\mathbf{P}| \rangle$ will decay along the wire. This is an old problem initiated by seminal (semiclassical) study of D'yakonov' and Perel (DP).¹⁵ We demonstrate the decay of spin coherence within our quantum transport formalism in Fig. 2, where the decay rate decreases in narrow wires thereby suppressing the DP mechanism.¹⁶ The effect acquires a transparent physical interpretation within the same framework invoked in Sec. 3—the spin decoherence is facilitated when there are more conducting channels to which spin can entangle in the process of spin-independent charge scattering that induces transitions between the channels. Moreover, we find in Figure 2 quantum corrections to spin diffusion in strongly disordered systems, which capture Rashba spin precession beyond the DP theory or weak localization corrections to it (that are applicable for weak SO correction in random potential that can be treated perturbatively¹⁶). The current spin polarization $\langle |\mathbf{P}| \rangle$ in the wires of fixed length is recovering with disorder as soon as the diffusive transport regime is entered. Within the picture of spin entangled to an effectively zero-temperature “environment” composed of orbital transport channels, this effect has a simple explanation for *arbitrary* disorder strength: as the disorder is increased, some of the channels become closed for transport thereby reducing the number of degenerate “environmental” quantum states that can entangle to spin.

Recently a lot of theoretical interest¹⁴ has been directed toward relaxing strict ballistic transport regime required in the original³ spin-FET. In non-ballistic spin-FET proposal, tuning of equal Rashba and Dresselhaus SO interaction $\alpha = \beta$ is predicted to cancel the spin randomization due to charge scattering. We confirm in Fig. 3 that spin coherence computed within our formalism indeed does not decay along such 2DEG wire. However, length-independent constant value of $\langle |\mathbf{P}| \rangle$ is set below one ($S > 0$) and, moreover, depends on the spin-polarization properties of injected current. Thus, transported in such specially crafted 2DEG environment will remain in partially coherent state, rather than being described by a single spin wave function. This can be traced to the observation of diminished spin coherence in clean systems [see panel (b) of Fig. 1].

5. Conclusions

In conclusion, we have shown that transported electron spin in multichannel semiconductor spintronic devices (such as spin-FET³ or spintronic rings^{5,17}) can be subjected to the loss of coherence due to an interplay of SO interactions and any type

of charge scattering (including the one at the lead-2DEG interface). Nonetheless, spins remain in a partially coherent¹⁸ state that can exhibit quantum interference effects of reduced visibility¹² [as indicated in panel (b) of Fig. 3].

Acknowledgments

We are grateful to F. E. Meijer, L. P. Zârbo, and S. Welack for valuable discussions.

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